STABILITY OF ELASTOPLASTIC FLOWS

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The fundamental inhomogeneity of plastic strain is a well-known fact; it can become concentrated in individual slip planes. In the case of sufficiently high strain rates, the strain-localization effect can result in fracture [1]. A reduction in temperature is similarly manifested by an increase in the strain rate [2].

We now investigate the stated problem from the standpoint of stability of elastoplastic flows.

We use the model of a viscoelastic fluid, which adequately describes the behavior of the material at high strain rates.

The motion of a viscoelastic medium is described by the equations

$$\rho\left(\frac{\partial u_i}{\partial t}+u_k\frac{\partial u_i}{\partial x_k}\right)=\frac{\partial \sigma_{ik}}{\partial x_k}; \tag{1}$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u_k)}{\partial x_k}=0; \tag{2}$$

$$\frac{\partial \varepsilon_{ik}}{\partial t}=\frac{1}{2\mu'}\left[\sigma_{ik}-\frac{\lambda'}{2\mu'+3\lambda'}\delta_{ik}\sigma_{pp}\right]+\frac{1}{2\mu}\left[\frac{\partial \sigma_{ik}}{\partial t}-\frac{\lambda}{2\mu+3\lambda}\delta_{ik}\frac{\partial \sigma_{pp}}{\partial t}\right]; \tag{3}$$

$$\frac{\partial \varepsilon_{ik}}{\partial t}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}\right); \tag{4}$$

$$\rho c\left(\frac{\partial T}{\partial t}+u_k\frac{\partial T}{\partial x_k}\right)=\frac{1}{2\mu'}\sigma_{ik}\left[\sigma_{ik}-\frac{\lambda'}{2\mu'+3\lambda'}\delta_{ik}\sigma_{pp}\right]+k\Delta T; \tag{5}$$

$$\overset{\mathrm{V}}{\varepsilon}_{ii}=0, \tag{6}$$

in which σ_{ik} is the stress tensor; ρ is the density of the medium; $\overset{\mathrm{V}}{\varepsilon}_{ii}$ is the spherical part of the viscous strain tensor; c is the specific heat; k is the thermal conductivity; μ, λ are the Lamé constants; u_i denotes the components of the velocity; T is the temperature; and μ', λ' are the temperature-dependent viscosity coefficients. The first term on the right in Eq. (5) is the work of viscous deformation.

To solve the flow-stability problem it is necessary to perturb the main motion described by Eqs. (1)-(6) and to analyze their behavior with time.

For the simplest case of plane "layered" flow (Fig. 1a) we have

$$\sigma_{3i}=\varepsilon_{3i}=u_3=u_2=0.$$

In the case of an incompressible medium and temperature-independent elastic coefficients, we have for our problem

$$\sigma_{11}=\sigma_{22}=\sigma_{33}=0,\ \varepsilon_{33}=\varepsilon_{22}=\varepsilon_{11}=0.$$

Also, we adopt an elementary temperature dependence of the viscosity coefficient: $\mu'=1/bT$ (b = const). Then (1)-(6) assume the following form for the components σ_{12} and u_1 (we drop the subscripts from now on):

$$\rho\partial u/\partial t=\partial\sigma/\partial y,\ \partial u/\partial y=(1/2\mu)\partial\sigma/\partial t+bT\sigma,\ \rho c\partial T/\partial t=k\partial^2T/\partial y^2+bT\sigma^2,$$

or, transforming to dimensionless variables

$$\tilde{\sigma}=\sigma/G;\ \tilde{T}=(bk/c)T;\ \tilde{y}=y\sqrt{(b/k)}G;\ \tilde{t}=(cG/k)t;\ G=2\mu,$$

we obtain

$$\frac{\partial\tilde{u}}{\partial\tilde{t}}=A\frac{\partial\tilde{\sigma}}{\partial\tilde{y}},\ \frac{\partial\tilde{u}}{\partial\tilde{y}}=\frac{\partial\tilde{\sigma}}{\partial\tilde{t}}+\tilde{T}\tilde{\sigma}, \tag{7}$$

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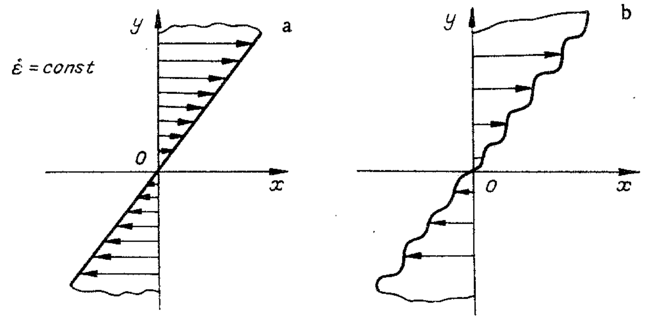

Fig. 1

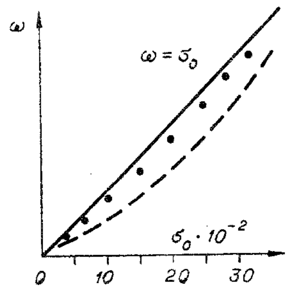

Fig. 2

where

$$\frac{\partial \tilde{T}}{\partial \tilde{t}} = A\frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2} + AT\sigma^2; \quad A = \frac{bGk}{\rho c^2}$$

(from now on we drop the tilde above the dimensionless variables).

The unperturbed flow with constant strain rate $\dot{\varepsilon}_0$ (variables with subscript 0) satisfies the equations

$$\frac{d\sigma_0}{dt} = \frac{du}{dy} - T_0\sigma_0, \quad \frac{dT_0}{dt} = AT_0\sigma_0^2. \tag{8}$$

Varying the system of equations (7), we obtain for the perturbations u', σ', and T'

$$\frac{\partial u'}{\partial t} = A\frac{\partial \sigma'}{\partial y}, \quad \frac{\partial \sigma'}{\partial t} = \frac{\partial u'}{\partial y} - T_0\sigma' - \sigma_0 T', \quad \frac{\partial T'}{\partial t} = A\frac{\partial^2 T'}{\partial y^2} + 2AT_0\sigma_0\sigma' + A\sigma_0^2 T'. \tag{9}$$

Strictly speaking, the system (8) does not have steady-state solutions. For small values of A, however, we can analyze the quasi-steady-state case ($d\sigma_0/dt = 0$). Then $T_0\sigma_0 = du/dy = \dot{\varepsilon}_0$, and the coefficients T_0 and σ_0 in the system of equations (9) are constants. The error introduced by the latter assumption is subsequently tested by direct numerical integration of the system (8), (9).

Seeking a solution of the system (9) with constant coefficients in the form

$$u' = A' \exp(i\omega y + \beta t),$$

$$\sigma' = B' \exp(i\omega y + \beta t), \quad T' = C' \exp(i\omega y + \beta t),$$

we obtain the characteristic equation

$$\beta^3 + A(\omega^2 + T_0 - A\sigma_0^2)\beta^2 + [A\omega^2(T_0 + 1) + A\sigma_0^2 T_0]\beta + A^2\omega^4 - A^2\omega^2\sigma_0^2 = 0.$$

The eigenvectors $\mathbf{R} = \{u, \sigma, T\}$ corresponding to the eigenvalues $\beta_i\omega_i$, have the form

$$\mathbf{R}_i = \left\{-\frac{A\sigma_0\omega}{A\omega^2 + T_0\beta_i + \beta_i^2}; \quad -\frac{\sigma_0\beta_i}{A\omega^2 + T_0\beta_i + \beta_i^2}; \quad 1\right\}.$$

For unstable perturbations $\mathrm{Re}\beta > 0$ (for at least one root). Consequently, invoking the Hurwitz condition, we arrive at the flow-stability condition

$$\omega < \sigma_0,$$

or, in dimensioned form, $\omega < (\dot{\varepsilon}/T)\sqrt{1/bk}$.

It is natural to suppose that the real ultimate flow pattern (see Fig. 1b) will be determined by perturbations having the maximum growth rate. Numerical computations based on Eq. (9) for the determination of such perturbations are represented by the dashed curve in Fig. 2; the solid line separates the instability domain ($\omega < \sigma_0$) from the stability domain; the dots indicate the locus of the boundary of the instability domain as obtained by simultaneous numerical solution of the systems (8) and (9), i.e., with allowance for the time variation of the coefficients T_0 and σ_0 in Eq. (9).

The foregoing results show that with an increase in the strain rate (or a decrease in the temperature) the distance between strain-localization planes diminishes (in conformity with the experimental data).

It is important to note that the conclusions reached above are of a qualitative theoretical character and are valid only for large strain rates. In particular, the conclusion about the existence of unstable perturbations for arbitrary small strain rates is not physical, because a different model of the medium is required in this case.

Moreover, in the general case it is required to solve the appropriate boundary-value problem, in which case appreciable changes can be incurred. The conclusions obtained above can only be valid for domains in which the characteristic space scale is much greater than the space scale of the unstable perturbations.

LITERATURE CITED

1. R. F. Recht, "Catastrophic thermoplastic shear," Trans. ASME, Ser. E: J. Appl. Mech., 86, No. 2, 189 (1964).
2. A. F. Brown, "Surface effects in plastic deformation of metals," Adv. Phys., 1, 427-479 (1952).

A TECHNIQUE FOR THE SOLUTION OF WAVE PROBLEMS FOR A NONLINEAR COMPRESSIBLE MEDIUM

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In this article we give analytical solutions of the problems of one-dimensional and two-dimensional stationary shock propagation in an ideal nonlinearly compressible medium under the action of sudden strong disturbances in the form of explosive impulses. We investigate the one-dimensional nonstationary problems of a plane and a spherical layer, and in the two-dimensional context solve the problem of the action of a moving disturbance (load) on an inelastic half-plane for the case in which the velocity D of a disturbance moving along its boundary is greater than the shock propagation velocity in the material of the half-plane. It is assumed in all the problems that the medium at the shock is subjected instantaneously to a nonlinear load and that linear irreversible loading takes place in the disturbed postshock region (Fig. 1). This statement of the problems permits them to be solved by the inverse route, i.e., by specifying a definite form (velocity) for the shock surface and determining the corresponding loading profile at the boundary of the layer or half-plane. In this case the motion of the medium in the unloading region is described by the wave equation in two variables, in application to which a Cauchy problem is formulated; it is known [1] that a solution of this problem exists and is unique. In a concrete example we examine the case in which the equation for the shock surface is given as a second-degree polynomial in ξ and we compare the results of the computations with results obtained on the basis of the method of characteristics [2], which yields satisfactory agreement of all the parameters of the medium.

The case of linear loading and unloading of the medium for the two-dimensional problem has been investigated in [3, 4]. A solution of the problem of the propagation of convergent spherical and cylindrical shocks in an ideal inelastic medium with rigid unloading is given in [5]. The investigated problems have potential practical applications in the study of strong disturbances in water-impregnated soils and in reservoirs.

§1. Propagation of One-Dimensional Plane and Spherical Shocks in a Nonlinearly Compressible Medium

Let a monotonically decreasing load $p_0(t)$ be applied to the boundary of a layer. As a result, a shock wave propagates in the medium with leading edge $r = R(t)$, behind which unloading takes place. In this case, for the disturbed region we have equations of motion, continuity, and state in the form

$$\frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \quad \frac{\partial \rho}{\partial t} + \rho\left(\frac{\partial u}{\partial r} + \frac{\nu u}{r}\right) = 0, \quad p(r, t) = p^* + E(\varepsilon - \varepsilon^*), \tag{1.1}$$

where $\varepsilon = 1 - \rho_0/\rho$, $E = c_p^2 \rho$. At the shock $r = R(t)$ we have relations of the form

$$u^*(t) = \varepsilon^* \dot{R}, \quad p^* = \rho^* \varepsilon^* \dot{R}^2, \quad p^*(t) = \alpha_1 \varepsilon^* + \alpha_2 \varepsilon^{*2} \quad (\dot{R} = dR/dt). \tag{1.2}$$

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